# Optimum design of a photovoltaic powered pumping system

Wagdy R. Anis<sup>a</sup> and M.A. Nour<sup>b, \*</sup>

<sup>a</sup>Electronics and Communication Department, Faculty of Engineering, Ain Shams University, 1 Sarayat Street, Abbasia, Cairo (Egypt) <sup>b</sup>Yanbu Industrial College, Electronics Departement, PO Box 30436, Yanbu Al Sinaiyah 21477 (Saudi Arabia)

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## Abstract

Photovoltaic (PV)-powered pumping systems are relatively simple and reliable. Hence, they are applied worldwide. Two conventional techniques are currently in use: the first is the 'directly-coupled' system where a PV array is directly coupled to a d.c. motor-pump group; the second is the 'battery-buffered' system where a battery is connected across the array to feed the d.c. motor that drives the pump. Recently, a third system has been proposed, namely, the 'switched-mode' PV-powered pumping system. This system couples the pump to the PV array directly when the storage battery is fully charged. The objective is the maximum utilization of available solar radiation to minimize the cost per pumped cubic meter from a given water depth. For a given location, four main parameters affect the design of this system: (i) d.c. motor-pump group parameters; (ii) PV array size; (iii) battery storage size, and (iv) water storage tank size. It is found that some of the factors are more effective in reducing the cost than others. The PV array size is the predominant factor, while the battery storage and water-tank sizes have relatively less effect. A detailed economic analysis is given.

## Introduction

Photovoltaic (PV)-pumping systems may be classified as follows:

(i) 'Directly-coupled' systems where the PV array is directly coupled to a d.c. motorpump group. Such a system is simple and reliable, but the motor does not operate continuously at its optimum operating point due to the continuous variation of solar irradiance. Previous studies have analysed both the steady-state [1-3] and the dynamic [4] performance of this system.

(ii) 'Battery-buffered' systems where a storage battery is connected across the PV array and the d.c. motor is operating at almost constant voltage (note that the battery voltage varies around its nominal value) and, as a result, the motor is operating at almost its optimum operating point. This system has two advantages over the directly-coupled one. First, water may be pumped day and night, thus water discharge is larger. Second, the d.c. motor is operating at its optimum operating point and consequently the system efficiency is enhanced. A major disadvantage of such a system is the extra system cost due to added battery cost. Thus, a battery-buffered system allows more water pumping, but at high system cost. Again, the criterion determining which system

<sup>\*</sup>Author to whom correspondence should be addressed.



Fig. 1. Schematic diagram of a PV-powered pumping system.

is better economically is the cost of a pumped cubic meter. The disadvantage of this system is the energy loss due to array disconnect when the battery is fully charged during high solar radiation periods [5].

(iii) Switched-mode PV pumping system which is designed to overcome the problem of the energy loss. This system uses a storage battery as a buffer between the PV array and the pumping system, but a novel battery voltage regulator is designed to connect the PV array directly to the pumping system when the storage battery is fully charged. Thus, the energy loss due to array disconnect is drastically reduced [5]. The block diagram of this system is given in Fig. 1.

The work reported here considers the design of a switched-mode PV-pumping system and discusses how different parameters influence the system design. The design objective is to determine the optimum conditions for minimum cost.

## System parameters

The principles of operation, as well as the modelling of the switched-mode PV pumping system, have been discussed previously [5]. The developed model takes into consideration the following factors:

- instantaneous variation of solar irradiance in Cairo city (30° N)
- effect of ambient temperature on the PV-array performance
- effect of both series and shunt resistors of PV array
- effect of charging and discharging current on battery voltage
- effect of the instantaneous state-of-charge on battery voltage
- effect of battery capacity on battery performance
- d.c. motor-pump group characteristics

To develop a general design, regardless of motor-pump size or the site under consideration, the following parameters are defined:

(i) The array factor  $(F_a)$  is given by:

$$F_{a} = \frac{\text{peak power of the PV array (kW)}}{\text{load power (kW)}}$$

 $F_a = P_p / P_l$ 

(1)

where  $P_p$  is the peak power (kW) of the array, and  $P_1$  the load power (kW). (ii) The storage factor  $(F_s)$  is given by:

 $F_{\rm s} = \frac{\text{battery storage capacity (kWh)}}{\text{daily load energy (kWh)}}$ 

## $F_{\rm s} = S/24P_{\rm l}$

where S is the battery storage expressed in kWh. The factor 24 represents the hours of the day. If both load power and daily load energy varies from day to day, then eqns. (1) and (2) consider the annual average daily load demand and the annual average daily load consumption, respectively. Note that,  $F_a$  and  $F_s$  are dimensionless since they are ratios. For the PV-pumping system, the designer may consider one of the following alternatives to achieve the ultimate goal of minimum cost.

• Continuous operation of the motor pump for 24 h. This choice requires a relatively large battery storage size and larger amount of pumped water. If the water demand is continuous, such a design will reduce the water storage tanks.

• Daily operation only. Hence, the water is pumped only whenever solar radiation is available. If the load demand is continuous, then more water has to be pumped during the day to supply the load demand during the night. Such a design necessitates a larger motor-pump group, a larger array size, and larger water storage tanks. On the other hand, this design does not require a large battery storage.

The preference between the two alternatives depends on: (i) load water demand throughout a complete year, and (ii) system economics – the minimum the cost of one cubic meter pumped, the better is the design. From the above discussion, it is necessary to define a water storage factor  $F_w$ , i.e.:

$$F_{\rm w} = \frac{\text{water tank storage size (m^3)}}{\text{daily load water demand (m^3)}} = \frac{S_{\rm w}}{bP_1}$$
(3)

The daily load water demand is proportional to load power  $P_1$ . The term  $S_w$  is the water storage tank size. In eqn. (3), b is a constant with units of  $(m^3/kW)$ . Again,  $F_w$  is a dimensionless parameter. Another factor to be considered to achieve the minimum cost is the pump installation cost. To express this parameter as a dimensionless factor, the pump installation cost parameter  $(F_p)$  is defined as:

$$F_{p} = \frac{\text{pump installation cost ($)}}{\text{PV-array cost ($)}}$$
(4)

This parameter depends mainly on the geology of the site where the pump is to be installed. If water is to be pumped from a well in a rocky area, the pump installation cost will be high, while if water is pumped from surface water, the pump installation will be low. The above-mentioned parameters allow the determination of the optimum design for a given situation.

### **Economic analysis**

Since the economics is the predominant parameter in system design, a unit cost function  $C_{u}$  can be defined, as follows:

 $C_{u} = \frac{\text{overall system cost during system lifetime}}{\text{total amount of pumped water during same period}}$ 

The total amount of daily pumped water is proportional to the load power  $P_1$ . If the overall system cost is denoted by  $C_T$ , then:

 $C_{\rm u} = C_{\rm T}/365NbP_{\rm H}$ 

(2)

(5)

N is the expected lifetime of the system (expressed in years). The factor 365 is simply the number of days per year.

Overall system cost includes: PV array cost including its installation; battery storage cost including wiring and battery voltage regulator cost; water storage tank costs including piping and installation costs, and operation and maintenance costs during a given period. The expected lifetime of the PV array is 20 years. Thus, the economic analysis given hereafter is based on a 20-year period. Battery lifetime is not expected to exceed 5 years for commercial industrial lead/acid batteries. The operation and maintenance costs include: the d.c. motor brushes change; storage battery maintenance (addition of distilled water or acid and monitoring of cell voltage), and water storage tank cleaning. Since most of the system cost is paid at the installation time, while the amount of pumped water is produced during the expected lifetime of the system, then the economic analysis should recognize this factor. The parameters to be considered are the money discount rate (d) and the inflation rate of battery storage  $(i_s)$ . The major constraint to the development of an economical model is that the load water demand should be met without interruption. For the sake of simplicity, the load water demand throughout the year is assumed to be constant both day and night. Thus, even during the cloudy days, the water tank should be capable of delivering the load water demand.

Assume that the expected lifetime of the PV array is N years (usually considered to be 20 years) and that of the battery storage is k years (usually considered to be 5 years). Thus, the cost of the storage battery is paid at the beginning of system installation (considered zero time) and partially paid each k years. If the cost of 1 kWh of storage batteries is  $C_s$ , then the present value of the storage battery replacement cost  $C_R$  is given by [6]:

$$C_{\rm R} = S\{C_{\rm s}[1+(i_{\rm s}-d)]^k + C_{\rm s}[1+(i_{\rm s}-d)]^{2k} + \dots C_{\rm s}[1+(i_{\rm s}-d)]^{N-k}\}$$

where S is the battery storage capacity expressed in kWh. Since, from eqn. (2),  $S = 24F_sP_1$ , one can write:

$$C_{\rm R} = 24F_{\rm s}P_{\rm l}C_{\rm s}[1+(i_{\rm s}-d)]^{k}\{[1+(i_{\rm s}-d)]^{N-k}-1\}/\{[1+(i_{\rm s}-d)]^{k}-1\}$$
(6)

The initial cost of the system  $C_i$  (paid at the installation time) is composed of the array cost, the cost of the initial storage battery (whose capacity is S), and the cost of the water storage tank. If the cost of one kW peak of the PV array is  $C_a$ ( $kW_p$ ), then the array cost is  $C_aP_p$ . Using eqn. (1), the array cost will be  $C_aF_aP_i$ . If the cost of water storage is  $C_w$  ( $m^3$ ) then, using eqn. (3), the water tank cost will be  $bF_wC_wP_i$ . Thus, the initial system cost is:

$$C_{i} = C_{a}F_{a}P_{i} + 24F_{s}P_{l}C_{s} + bF_{w}C_{w}P_{l} + C_{ins}$$
<sup>(7)</sup>

The last term,  $C_{ins}$ , in eqn. (8) is the pump installation cost. From eqns. (1) and (4):

$$C_{\rm ins} = F_{\rm p} F_{\rm a} C_{\rm a} P_{\rm l} \tag{8}$$

The total system cost  $C_{\rm T}$  is:

$$C_{\rm T} = C_{\rm i} + C_{\rm R} \tag{9}$$

Combining eqns. (5) through (9) yields:

$$C_{u} = (1/365Nb) \{C_{a}F_{a}(1+F_{p}) + 24F_{s}C_{s} + bF_{w}C_{w} + 24F_{s}C_{s}[1+(i_{s}-d)]^{k} \{[1+(i_{s}-d)]^{N-k}-1\}/\{[1+(i_{s}-d)]^{k}-1\}\}$$
(10)

Equation (10) gives the cost per unit in terms of the system parameters  $F_a$ ,  $F_s$ ,  $F_p$  and  $F_w$ . The other parameters in eqn. (10) are the unit costs of PV array  $C_a$ , battery storage  $C_{ss}$ , water storage tank  $C_w$ , and installation  $F_p$ .

#### Water tank size

It is assumed that the load water demand is constant both day and night throughout the year. The load water demand will be denoted by  $Q_{\rm L}$  (m<sup>3</sup>/day). The system pumps a varying amount of water according to the available solar radiation and the energy stored in the batteries. For a PV-pumping system, three parameters may be identified for the pumped water: annual average daily water discharge  $Q_{\rm a}$ ; maximum daily water discharge  $Q_{\rm max}$ , and minimum daily water discharge  $Q_{\rm min}$ . The previously mentioned water discharge parameters are computed on annual base and their units are (m<sup>3</sup>/day). Since the load requires  $Q_{\rm l}$  (m<sup>3</sup>/day) and if the amount of pumped water by the system is smaller than  $Q_{\rm L}$ , then the difference should be obtained from water tank. The worst condition takes place during the cloudy days where the system depends mainly on the storage energy in the batteries. During cloudy days, the system's water discharge is  $Q_{\rm min}$ . If the number of successive cloudy days is *m* then the minimum tank storage size  $(S_{\rm w})_{\rm min}$  is:

$$(S_{\rm w})_{\rm min} = (m+1)(Q_{\rm L} - Q_{\rm min}) \tag{11}$$

In eqn. (11), the term (m+1) is used instead of *m* because the period of cloudy days starts with the night of the previous sunny day and ends with the night of the last cloudy day, hence, an extra day of storage is required. Actually, a cloudy day represents about 12 h, while the night before and night after represent almost 24 h. Therefore, the duration of insufficient solar energy for a single cloudy day is 36 h or 1.5 days. For *m* cloudy days, there should be sufficient stored energy or water for (m+0.5) days. For conservative design, this factor is taken as (m+1).

The load demand determines the size of the d.c. motor-pump to be used in the system. If the rated discharge of the group is  $Q_0$  (m<sup>3</sup>/h), then the load demand should not exceed 24 $Q_0$  (m<sup>3</sup>/day). Practically, when the load demand is determined, the motor-pump group rating is computed from the following inequality:

$$Q_0 > Q_L/24$$
 (12)

Note that the factor 24 rises from the fact that  $Q_0$  is the hourly discharge while  $Q_L$  is the daily load water demand. To convert the inequality into an equation, a factor of safety R (greater than unity) is assumed, i.e.:

$$Q_0 = Q_L R/24 \tag{13}$$

The safety factor R is used because the motor-pump group cannot operate at its optimum operating point continuously due to battery voltage and temperature variations.

To determine the load power  $P_1$  (expressed here in W) required, it is necessary to know the overall water head h(m) of the system. This head is the sum of the water depth and the water tank height. The load power is related to the overall water head h and the rated discharge  $Q_0$  (expressed here in  $m^3/s$ ) through:

$$\eta P_1(\mathbf{W}) = Q_0 \ (\mathbf{m}^3/\mathbf{s}) \ \rho gh \tag{14}$$

where:  $\eta$  is the pump efficiency;  $\rho$  the water density (100 kg/m<sup>3</sup>), and g the earth's gravitational acceleration (9.8 m/s<sup>2</sup>). If it is required to keep  $P_1$  in kW and  $Q_0$  in

 $m^{3}/h$  then a constant *a* has to be added in eqn. (14), i.e.:

$$\eta P_1$$
 (kW) =  $aQ_0$  (m<sup>3</sup>/h)  $\rho gh$ 

Eqns. (13) and (15) enable the designer to determine both  $P_1$  and  $Q_0$ .

#### Interdependence of system parameters

Consider that a load demands 100 m<sup>3</sup> per day and that the water is pumped from a well of 25 m depth. Assume that the water tank height is 5 m, then the overall water head of the pump is 30 m. Further assume that R=2, then eqn. (13) gives  $Q_0 = 8.33$  m<sup>3</sup>/h. Making use of eqn. (14) and for 80% pump efficiency,  $P_1 = 868$  W. A d.c. motor operating at 48 V and 18 A would be a reasonable choice. The forthcoming analysis is based on this selected system. It is necessary to determine the optimum array, battery storage and water tank sizes that meet the load demand at minimum cost. The pump installation factor  $F_p$  is considered to be 0.15; this means that the installation cost is 15% of the PV-array cost. The water storage factor  $F_{\rm w}$  – according to eqns. (3) and (11) – depends on the number of successive cloudy days at the site under consideration (m) and the battery storage and array sizes that determine the amount of pumped water during cloudy days ( $Q_{min}$ ). For Cairo city (30° N), the number of successive cloudy days is considered to be 2; i.e., m=2. If the battery and array sizes are designed to supply the load water demand for 2 successive days, then a water storage tank of minimum size may be sufficient. The expected PV-array lifetime is 20 years (i.e., N=20), while the storage battery lifetime is 5 years only (i.e., k=5). The money discount rate is considered to be 10% (i.e., d=0.1) and storage battery inflation rate  $i_s$  to be 12%. The constant b is estimated to be 0.1. The cost of 1 kW peak of PV array ( $C_a$ ) is about US\$ 5000. The storage cost  $C_s$  is about US\$ 50 per kWh of storage batteries. The cost of a water tank is considered to be US\$ 25 per m<sup>3</sup>, i.e.,  $C_w = 25$ . The remaining two parameters are the array factor  $F_a$  and the storage factor  $F_s$ . It is seen from eqn. (10) that the cost per unit  $C_u$  increases as  $F_a$  or  $F_s$ increases. Thus, the optimum economical solution is obtained at the minimum values of  $F_{a}$  and  $F_{s}$  that satisfy the load requirements. Note that there is an interdependence between the water tank size and array and battery sizes. This fact may be explained as follows, as battery and array sizes increase the amount of pumped water during cloudy days increases, hence  $Q_{\min}$  increases and consequently  $(S_w)_{\min}$  decreases as seen from eqn. (11). The increase in battery storage and array size leads to a decrease in water tank size only to a certain limit. The reason is that if battery storage and array are increased to the extent that the load energy is totally supplied by the storage battery during the cloudy days, then  $Q_{\min} = Q_L$  and from eqn. (11) it can be deduced that the water tank size becomes zero. Table 1 gives 10 groups of system parameters that satisfy the load requirement. It is observed from Table 1 that as  $F_a$  and  $F_s$  increase,  $F_{w}$  decreases as expected. The minimum cost is obtained at the minimum value of  $F_{\rm a}$  since the array cost is the dominant factor in the cost analysis.

#### **Results and analysis**

To obtain the optimum design, all system parameters are kept constant except one parameter only. Thus, a family of curves are obtained. Figure 2 shows the system water discharge for a constant storage factor  $F_s$  and different array factors during a

## TABLE 1

System	parameters	that	satisfy	the	load	demand	and	the	associated	cost	in	each	case	
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Fa	$F_{s}$	$F_{w}$	$C_{\rm u}$ (cent/m <sup>3</sup> )
2.83	0.37	1.80	2.51
3.54	0.44	1.70	3.12
4.55	0.84	1.09	4.23
5.05	0.95	0.85	4.71
5.56	1.15	0.78	5.26
6.06	1.26	0.75	5.74
6.57	1.47	0.59	6.30
7.07	1.57	0.40	6.77
7.58	1.68	0.19	7.26
7.38	1.76	0.00	7.52



Fig. 2. Daily water discharge (fixed storage factor).

complete year. It is clear that as  $F_a$  increases, the amount of pumped water increases significantly. The directly-coupled water discharge is presented in Fig. 3. It can be seen that the discharge is affected appreciably by the array factor. Figure 4 gives the effect of the battery storage factor  $F_s$  on the amount of pumped water for a constant array factor. It is clear that the storage factor has a minor effect on water discharge. Thus, the storage factor should be kept at the minimum value that satisfies the system design constraint. Figure 5 indicates that the effect of the storage factor on the system performance is that when  $F_s$  increases, the state-of-charge of the battery is kept generally higher, except during the cloudy days where all curves coincide. Of course when the state-of-charge is kept high, the battery lifetime is prolonged. Nevertheless, this advantage does not justify the enlarging of the battery storage factor. To check if given values of  $F_a$ ,  $F_w$  and  $F_s$  satisfy the load requirement, a detailed simulation program for a complete year must be run. The program gives the daily water discharge;



Fig. 3. Directly-coupled daily water discharge (fixed storage factor).



Fig. 4. Total daily power discharge (fixed array factor).

if it is larger than the load requirement throughout the year – without a single day exception – then the solution is accepted, otherwise the solution is refused. Table 1 indicates that the optimum parameters are:  $F_a = 2.83$ ,  $F_s = 0.37$  and  $F_w = 1.8$ .

This optimum solution results in a water-pumping cost of 2.65 cents per cubic meter pumped from a depth of 30 m.



Fig. 5. Final daily state-of-charge (SOC) of the battery (fixed array factor).

## Conclusions

The optimum design of a PV-powered pumping system that minimizes the cost of pumping one cubic meter has to consider three basic parameters, namely, PV array size, battery storage size and water tank size. It is found that the optimum solution is one that minimizes the PV array size. This is because the array cost is the major item. Increasing the battery storage without increasing array size has little effect on system performance. The designer has to choose one of two alternatives: either to increase the water tank size or to increase both array and battery storage sizes simultaneously. The former choice is the most economical.

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